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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The Report summarizes theoretical research on internal atmospheric waves, on the self-effect in electro-magneto-elasticity, on the mechanics of ferromagnetic materials, on viscoelasticity, on compensated compactness for nonlinear partial differential equations, on peristaltic pumping with heat release and heat conduction, and on sheet flow of a magnetic viscous fluid. Keywords: Solid mechanics, visco-magnetic fluid sheets. (KR) ←

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Final Technical Report
on Grant No. AFOSR-87-0194

This Report summarizes research funded partially by the Grant as follows:

Publications

R. E. L. Turner and J.-M. Vanden-Broeck, Interfacial solitary waves, Phys. Fluids, in press.

J. W. Robbin, R. C. Rogers and B. Temple, On weak continuity and the Hodge decomposition, Trans. A.M.S. 302, 1987.

R. C. Rogers and B. Temple, A sufficient condition for weak continuity of polynomials in the method of compensated compactness, Trans. A.M.S., in press.

J. A. Nohel, R. C. Rogers and A. E. Tzavaras, Weak solutions for a nonlinear system in viscoelasticity, Commun. Partial Diff. Eqs., 13, 1988.

R. C. Rogers, Nonlocal variational problems in nonlinear electro-magneto-elasticity, SIAM J. Math. Anal., in press.

Several further publications are in preparation.

Robert E. L. Turner

Internal Waves in Stratified Atmospheres

Work was continued on the study of internal waves in stratified fluids using both analytical and computational techniques. Attention was particularly directed to waves in a two-fluid system, confined between horizontal walls and acted on by gravity. The underlying equations are the Euler equations for nondiffusive flow. For each base configuration of fluid depths and densities it was shown by Amick and Turner in [1] that there are global branches of solitary wave solutions starting with infinitesimal waves corresponding to a critical speed for the system. Open questions regarding the limiting form of these waves led to various projects [2],[3],[4] which were continued or completed during the grant period.

Amick and Turner conducted analytical investigation of all small waves in the two fluid system, particularly surges. A surge is a flow which approaches different limiting parallel flows as the horizontal coordinate approaches infinity to the left and to the right. No rigorous analytical studies of surges had been presented in the literature before, and now in [2] we have a complete description of all small amplitude interfaces between two irrotational regimes of differing densities under the influence of gravity. We used a dynamical systems approach and a center manifold reduction to model the behavior of small waves. In this way we could parametrise all 'small' solutions of the full system, a quasi-linear elliptic problem arising from the Euler equations through suitable changes of coordinates, by means of solutions of an ordinary differential equation of second order in a center manifold. In fact, the existence of an invariant for the flow allows a final reduction to a first order differential equation, the heteroclinic orbits for which correspond to surges. The work for this project led to the broader study of functional equations of the type arising in hydrodynamics. In [3] we investigate mappings in weighted Hölder spaces which are useful for the analysis of a general class of functional equations. These equations include those arising in the internal wave problem as well as those from surface wave problems with surface tension [5] and, we expect, will be useful in other wave problems.

A project completed under the grant was a numerical study [4], done jointly with Jean-Marc Vanden-Broeck, to determine the limiting forms possible for the waves shown to exist in the paper [1]. From that paper we knew that along a branch of solutions the solitary waves would have to broaden indefinitely or the slopes of the streamlines in the flow would have to steepen approaching the vertical. The problem was recast in terms of linked, nonlinear integrodifferential equations in one variable parametrising the interface. The equations were discretized and the resulting nonlinear equations solved using the Newton-Raphson method. It was shown that for certain ranges of the density and depth parameters the solitary waves appeared to broaden indefinitely. For example, with density ratios like that of air and water and with equal fluid depths, broad waves were seen. This

was evidence for the existence of surges, a surge being roughly 'half' of an extremely broad wave.

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3. Amick, C. J. and Turner, R. E. L., Center Manifolds in Equations from Hydrodynamics, in preparation.
4. Turner, R. E. L. and Vanden-Broeck. J.-M., Interfacial solitary waves, Physics of Fluids, to appear.
5. Amick C. J. and Kirchgassner, K., Solitary water waves in the presence of surface tension, Arch. Rat. Mech. Anal., to appear.

Summary of Scientific Progress

Robert C. Rogers, Postdoctoral Associate

1 Self-Effect Problems in Electro-Magneto-Elasticity

Professor Rogers' work in [6] and with S.S. ANTMAN in [1] is part of a program to extend the modern existence and regularity theory of elasticity to problems in steady-state electromagnetism. These works examine problems involving polarizable, magnetizable, conducting, deformable materials with monotone electromagnetic response. The constitutive equations are assumed to have the general form:

$$\begin{aligned} \mathbf{T}(\mathbf{x}) &= \check{\mathbf{T}}(\mathbf{F}(\mathbf{x}), \mathbf{e}(\mathbf{x}), \mathbf{h}(\mathbf{x}), \mathbf{x}), \\ \mathbf{p}(\mathbf{x}) &= \check{\mathbf{p}}(\mathbf{F}(\mathbf{x}), \mathbf{e}(\mathbf{x}), \mathbf{h}(\mathbf{x}), \mathbf{x}), \\ \mathbf{m}(\mathbf{x}) &= \check{\mathbf{m}}(\mathbf{F}(\mathbf{x}), \mathbf{e}(\mathbf{x}), \mathbf{h}(\mathbf{x}), \mathbf{x}), \\ \mathbf{j}(\mathbf{x}) &= \check{\mathbf{j}}(\mathbf{F}(\mathbf{x}), \mathbf{e}(\mathbf{x}), \mathbf{h}(\mathbf{x}), \mathbf{x}). \end{aligned} \tag{1.1}$$

Here $\mathbf{T}$, $\mathbf{p}$, $\mathbf{m}$, and $\mathbf{j}$ are the Second Piola-Kirchhoff stress and the Lagrangian forms of the polarization, magnetization, and electric current respectively; and $\mathbf{F}$, $\mathbf{e}$, and $\mathbf{h}$ are the deformation gradient and the Lagrangian versions of the electric and magnetic fields respectively. These coupled, nonlinear relations allow for such effects as electrostriction and magnetostriction.

A suitable form of the Biot-Savart law implies that the magnetic field generated by the conducting, magnetized body satisfies the following integral equation:

$$\mathbf{h}(\mathbf{z}) = \nabla\phi(\mathbf{z}) + \int_\Omega \frac{\check{\mathbf{j}}(\mathbf{F}(\mathbf{x}), \mathbf{e}(\mathbf{x}), \mathbf{h}(\mathbf{x}), \mathbf{x})(\mathbf{z}-\mathbf{x})}{|\mathbf{z}-\mathbf{x}|^3} dz. \tag{1.2}$$

In [1] it was shown that under appropriate hypotheseson $\check{\mathbf{j}}$, $\mathbf{h}$ has the representation

$$\mathbf{h}(\mathbf{x}) = \nabla\psi(\mathbf{x}) + \kappa(\mathbf{F}, \mathbf{e}, \nabla\psi, \mathbf{x}), \tag{1.3}$$

where ψ is called the *magnetic potential* and $(\mathbf{F}, \mathbf{e}, \nabla\psi) \mapsto \kappa(\mathbf{F}, \mathbf{e}, \nabla\psi, \cdot)$ is compact on appropriate L^p spaces. Composing (1.3) with the constitutive equations (1.1), one can write the balance laws of elasticity and electro-magneto-statics in the form:

$$\begin{aligned} \operatorname{Div} \hat{\mathbf{T}}(\mathbf{F}(\cdot), \nabla\phi(\cdot), \nabla\psi(\cdot), \mathbf{F}(\mathbf{x}), \nabla\phi(\mathbf{x}), \nabla\psi(\mathbf{x}), \mathbf{x}) &= -\mathbf{f}(\mathbf{x}), \\ \operatorname{Div} \hat{\mathbf{d}}(\mathbf{F}(\cdot), \nabla\phi(\cdot), \nabla\psi(\cdot), \mathbf{F}(\mathbf{x}), \nabla\phi(\mathbf{x}), \nabla\psi(\mathbf{x}), \mathbf{x}) &= \sigma(\mathbf{x}), \\ \operatorname{Div} \hat{\mathbf{b}}(\mathbf{F}(\cdot), \nabla\phi(\cdot), \nabla\psi(\cdot), \mathbf{F}(\mathbf{x}), \nabla\phi(\mathbf{x}), \nabla\psi(\mathbf{x}), \mathbf{x}) &= 0. \end{aligned} \tag{1.4}$$

Here ϕ is the *electric potential*, and the definitions of $\mathbf{d}$ and $\mathbf{b}$ and $\mathbf{f}$ can be formed from $\mathbf{e}$, $\mathbf{h}$, and $\mathbf{F}$ in the usual way.

The existence of a large class of solutions for the system (1.4) was shown in [1] using techniques of pseudo-monotone operators (*cf.* [11]). The key observation is that the constitutive equations $\hat{\mathbf{T}}$, $\hat{\mathbf{d}}$, and $\hat{\mathbf{b}}$ are compact in the global variables and monotone in the local variables. In [6] a more general class of boundary conditions was considered, but greater restrictions were put on the constitutive equations.

2 Modeling of Ferromagnetic Materials

Professor Rogers has made several advances in the theory of ferromagnetic materials. In [6] the classical micromagnetic model was investigated, and an existence theorem was proved for coupled, unshielded, magneto-elastic materials under very general boundary conditions. The proof of the theorem involves methods of convex analysis and weak continuity. Nonlocal, shape-dependent effects are taken into account.

In addition, new models of ferromagnetic materials have been formulated and studied. In [2] a nonlocal constitutive relation is considered, and an existence theorem is proved using pseudo-monotone operator methods.

In [8] a nonlocal energy model is studied. The new model allows for the consideration of measure-valued magnetizations, which are suggested by recent studies of ERICKSEN [15], CHIPOT & KINDERLEHRER [12], and BALL & JAMES [9] on twinning in crystals. The energy of a ferromagnetic body with magnetization $\mathbf{m}$ in an applied field $\mathbf{h}_0$ is given by

$$\mathcal{E}(\mathbf{m}) = \frac{1}{2}\int_{\mathbf{R}^3} |\mathbf{h}_r(\mathbf{m})|^2 dv + \int_\Omega \{W(\mathbf{m}) - \mathbf{m}\cdot\mathbf{h}_0\}\, dv + \chi(\mathbf{m}). \tag{2.1}$$

The first integral is the *field energy*; $\mathbf{h}_r(\mathbf{m})$ is the resultant magnetic field induced by the magnetization $\mathbf{m}$. In the second integral, the first term is the *magnetization energy density*. We assume that W is nonconvex. In an isotropic material we expect W to be minimized for some prefered nonzero modulus of magnetization $\mathcal{M}_0$. The second term in the integral is the *interaction energy*,and the final term χ is the *exchange energy*. In the theory of micromagnetics an exchange energy of the form

$$\chi_1(\mathbf{m}) = \epsilon\int_\Omega |\nabla\mathbf{m}(\mathbf{x})|^2 dv_x \tag{2.2}$$

is used. In [8] the exchange energy is modeled by a nonlocal term of the form

$$\chi_2(\mathbf{m}) = C(\gamma)\left(\int_\Omega\left[\int_\Omega \frac{(\mathbf{m}(\mathbf{x})-\mathbf{m}(\mathbf{y}))e^{-\gamma|\mathbf{x}-\mathbf{y}|}}{|\mathbf{x}-\mathbf{y}|}dv_y\right]^2 dv_x\right)^2. \tag{2.3}$$

The new model for the exchange energy is designed to penalize local deviations in the magnetization while allowing for consideration of a wider class of magnetizations than that implied by an energy of the form (2.2). In particular, one can consider measure-valued distributions of magnetization. The use of such distributions is motivated by the work on twinning in crystals cited above.

If no exchange energy is present, the following oscillating sequence of magnetizations minimizes the energy absolutely for $-\mathcal{M}_0/3 \le \mathcal{H}_0 \le \mathcal{M}_0/3$:

$$\mathbf{m}^j = \mathcal{M}_0 f_{(\frac{1}{2}+\frac{3\mathcal{H}_0}{2\mathcal{M}_0})}(\frac{\mathbf{x}\cdot\mathbf{i}}{j})\mathbf{k}, \quad \mathbf{x}\in B_1(0), \tag{2.4}$$

Here $\mathbf{i}$ is a unit vector perpendicular to $\mathbf{k}$ and f_θ is a function of period 1 defined on the real line for $0 \le \theta \le 1$ by

$$f_\theta(x) = \begin{cases} 1 & 0 \le x \le \theta, \\ -1 & \theta < x < 1. \end{cases}$$

The sequence $\mathbf{m}^j$ does not converge strongly in L^2 but does converge weakly to a uniform "average" magnetization:

$$\mathbf{m}^j \rightharpoonup \frac{\mathcal{H}_0}{3}\mathbf{k}$$

Note that the average magnetization does not solve the original problem. (It solves what is known as the *relaxed* variational problem.) The idea introduced in the references on twinning cited above, is that one can think of the *Young's measure* of the minimizing sequence as solving the problem. Essentially, the Young's measure describes the oscillations of a weakly converging sequence. For instance, the Young's measure associated with the sequence $\mathbf{m}^j$ would be

$$\nu = \left(\frac{1}{2}+\frac{3\mathcal{H}_0}{2\mathcal{M}_0}\right)\delta(\mathcal{M}_0\mathbf{k}) + \left(\frac{1}{2}-\frac{3\mathcal{H}_0}{2\mathcal{M}_0}\right)\delta(-\mathcal{M}_0\mathbf{k}).$$

Here $\delta(\mathbf{v})$ is a Dirac measure centered at $\mathbf{v}$. The measure ν reflects the probability finding the values of the sequence $\mathbf{m}^j$ in either of the states $\pm\mathcal{M}_0\mathbf{k}$ in any small ball in $B_1(0)$. The center of mass of the measure is equal to the weak limit of the sequence, the average magnetization.

To see how the exchange energy χ_2 penalizes oscillations we examine how it is affected by the sequence $\mathbf{m}^j$. Some routine calculations give us the result

$$\lim_{j\to\infty}\chi_2(\mathbf{m}^j) = K\left[\left(\frac{1}{2}+\frac{3\mathcal{H}_0}{2\mathcal{M}_0}\right)(\mathcal{M}_0-\mathcal{H}_0/3)^2 + \left(\frac{1}{2}-\frac{3\mathcal{H}_0}{2\mathcal{M}_0}\right)(\mathcal{M}_0+\mathcal{H}_0/3)^2\right], \tag{2.5}$$

where K is a constant dependent on γ. Thus, χ_2 penalizes the *variance* of the Young's measure.

The point of view taken in the studies of twinning mentioned above that Young's measures can be thought of as solutions of the minimization problem. (This point of view descends directly from the ideas of generalized curves of L.C. YOUNG.) In [8] it is shown that the introduction of the nonlocal exchange energy leads to multiple measure-valued minima of the energy.

In [7] the following model problem was examined order to get a further idea of the effects of penalizing the variance of a measure-valued solution. Let $\mathbf{u} = (u_1, \ldots, u_n)$ be and element of $\mathbf{R}^n$. The following energy was minimized

$$\mathbf{E}(\mathbf{u}) = \sum_l \frac{\mathcal{W}(u_l)}{n} + \frac{\alpha_2}{2}\left(\sum_l \frac{u_l}{n}\right)^2 + \frac{\alpha_3}{2n}\sum_l \left(u_l - \sum_j \frac{u_j}{n}\right)^2 - \alpha_4 \mathcal{H}_0 \sum_l \frac{u_l}{n}. \tag{2.6}$$

Here $\mathcal{W}$ is the nonconvex energy

$$\mathcal{W}(v) = \begin{cases} \frac{\alpha_1}{2}(v - u_0)^2 & v \geq 0, \\ \frac{\alpha_1}{2}(v + u_0)^2 & v < 0, \end{cases} \tag{2.7}$$

$$= \frac{\alpha_1}{2}(v - \text{ sgn }(v) u_0)^2, \tag{2.8}$$

and $u_0, \mathcal{H}_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are positive constants. The first term is analogous to the magnetization energy, the second to the field energy, the third to nonlocal exchange energy, and the fourth to the interaction energy with $\mathcal{H}_0$ representing the strength of the applied field. Figure 1 represents the minimizers for $n = 8$. We

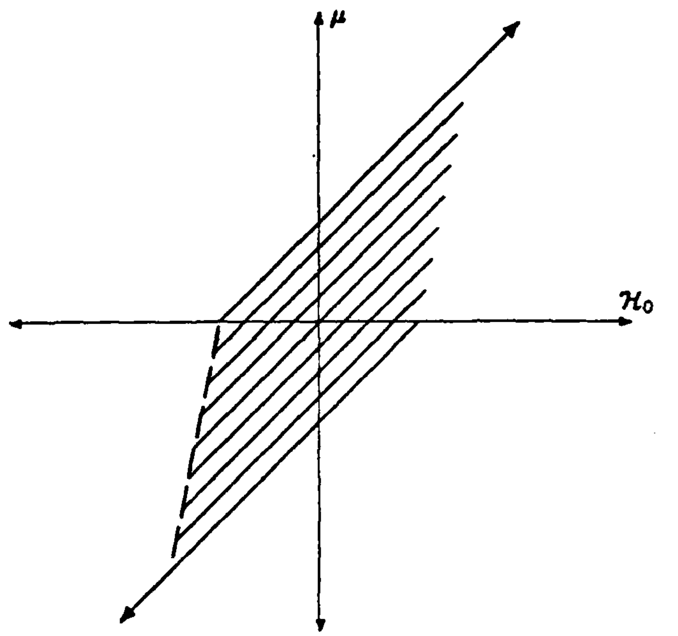

Figure 1: Solution curves for $\mathbf{u} \in \mathbf{R}^8$. Solution segments lie between the dashed lines $\mu = (\alpha_4\mathcal{H}_0 \pm \alpha_1 u_0)/(\alpha_2 - \alpha_3)$. All solution segments are parallel. The upper and lower half lines of solutions are on the lines $\mu = (\alpha_4\mathcal{H}_0 \pm \alpha_1 u_0)/(\alpha_1 + \alpha_2)$.

have graphed $\mu = \sum_i u_i$ which we think of as analogous to the center of mass of the Young's measure as a function of $\mathcal{H}_0$.

Raising α_3 (the coefficient of the moment term, our model for the exchange energy) causes the boundary lines to become more vertical. If we take the point of view that as $\mathcal{H}_0$ is slowly changed we will stay on a solution curve until forced to change by contact with the boundary, a more vertical boundary causes the inner loops of solutions to be much less stable than the outermost loop, thus making the solutions more like the hysterysis diagram of a hard ferromagnetic material.

3 Problems in Viscoelasticity

Recent work of Professor Rogers with J.A. NOHEL and A.E. TZAVARAS [5] initiated a program of investigating the existence of global weak solutions, in the class of bounded measurable functions, for the Cauchy problem for

$$\begin{aligned} w_t &= v_x, \\ v_t &= \sigma_x, \end{aligned} \qquad x \in \mathbf{R},\ t > 0, \tag{3.1}$$

with initial conditions

$$w(x,0) = w_0(x), \ \ v(x,0) = v_0(x), \quad x \in \mathbf{R}. \tag{3.2}$$

Here the function $\sigma(x,t)$ is determined by the history of $w(x,\cdot)$ through the constitutive assumption

$$\sigma(x,t) = \varphi(w(x,t)) + \int_0^t k(t-\tau)\psi(w(x,\tau))d\tau. \tag{3.3}$$

The given functions $\varphi(w), \psi(w)$ and $k(t)$ are assumed to be smooth and, in addition,

$$\varphi'(w) > 0, \quad w \in \mathbf{R}, \tag{3.4}$$

so that the structure of (3.1) is hyperbolic. This equation is a widely accepted model for important phenomena in viscoelastic solids.

In [5] the important special case $\psi \equiv \varphi$, is considered when the data $w_0, v_0 \in L^\infty(\mathbf{R}) \cap L^2(\mathbf{R})$. The problem becomes

$$\begin{aligned} w_t &= v_x, \\ v_t &= \varphi(w)_x + \textstyle\int_0^t k(t-\tau)\varphi(w(\cdot,\tau))_x d\tau, \end{aligned} \qquad x \in \mathbf{R},\ t > 0, \tag{3.5}$$

$$w(x,0) = w_0(x), \ v(x,0) = v_0(x), \quad x \in \mathbf{R}.$$

The constitutive function φ is assumed to satisfy

$$\left\{ \begin{array}{l} \varphi : \mathbf{R} \to \mathbf{R} \text{ is a twice continuously differentiable} \\ \text{function such that } \varphi'(w) > 0, \ \ w \in \mathbf{R}; \\ \varphi \text{ has a single inflection point at } w = w_i \text{ and is} \\ \text{convex on } (w_i, \infty) \text{ and concave on } (-\infty, w_i). \end{array} \right. \tag{3.6}$$

The kernel k satisfies

$$k : [0,\infty) \to \mathbf{R}, \quad k \in C^1[0,\infty), \tag{3.7}$$

and the data $w_0(x), v_0(x)$ satisfy

$$w_0(x), \ v_0(x) \in L^\infty(\mathbf{R}) \cap L^2(\mathbf{R}). \tag{3.8}$$

The following theorem was proved.

Theorem 3.1 *Let the hypotheses (3.6)-(3.8) be satisfied. Given $T > 0$, there exists a weak solution $\{w(x,t), v(x,t)\}$ of (3.5) on $\mathbf{R} \times [0,T]$, such that*

$$(w,v) \in L^\infty([0,T]; L^2(\mathbf{R})) \cap L^\infty(\mathbf{R} \times [0,T]). \tag{3.9}$$

DIPERNA [13] established similar results in the case of 1-d elasticity and 1-d isentropic gas dynamics.

4 Problems in Compensated Compactness and Weak Continuity

The techniques of compensated compactness and the characterization of weakly continuous functionals under various differential constraints has been an important tool in some recent developments in partial differential equations. In particular, the Div-Curl Lemma was instrumental in the work of TARTAR [16] and DIPERNA [13] on conservation laws and in our work on weak solutions in viscoelasticity [5]; and the characterization

of the null Lagrangians in the setting of the calculus of variations (*cf.* EDELEN [14]) was central to the work of BALL [10] on existence theorems in three-dimensional elasto-static polyconvex functions.

Professor Rogers, in work with J. ROBBIN and B. TEMPLE [3] devised a method sufficient for constructing weakly continuous polynomials via wedge products. This method produces polynomials of degree as high as the domain space, the maximum degree possible. With this method one can reproduce all of the weakly continuous functions in the div-curl case, the case of Maxwell's equations of electrodynamics, and the variational case. As Tartar indicates [17], the Quadratic Theorem can be used to prove such a theorem when L^2 techniques apply. The proof in [3] is based on the idea of decomposing weakly convergent sequences into weakly convergent and strongly convergent parts using a version of the Hodge decomposition. The result is more general than that obtained directly from the L^2 theory. In addition, the proof gives additional physical insight into the phenomenon of weak continuity and the interaction of oscillations.

The work on wedge products led to a more general sufficient condition for the weak continuity of polynomials. The condition (given in [4]) relates weak continuity to the wave cone and characteristic set of the differential constraints. It is both necessary and sufficient when the polynomial is quadratic or when a certain orthogonality condition is satisfied. Since the condition is more general than the wedge product condition it applys to all of the classical examples of weakly continuous polynomials.

This condition clarifies the relationship between the directions in the wave cone (associated with the dependent variables of the weakly convergent sequence) and the characteristic directions (associated with the independent variables of the sequence). The Legendre-Hadamard condition involves only the wave cone, but for degree greater than two, the coupling of these directions to the characteristic directions is crucial.

The method of proof for the general condition is to obtain conditions under which one can iterate the quadratic theorem of Tartar. In order to do this we characterize fully the first-order differential constraints induced in quadratics by first-order differential constraint on their factors.

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Peristaltic Pumping with Heat Release and Heat Conduction

M. C. Shen and Dalin Tang

Peristalsis was originally a physiological pumping mechanism which transports liquid along a duct by a large-amplitude, wavelike motion of the duct walls; the esophagus, urethelia and urethra work that way. It was picked up by medical engienering for antiseptic liquid transport and by nuclear engineering, because it allows radioactive liquid to be transported without corrosion of the pumping machinery. In the latter context, in particular, radioactive heat release and heat transfer become important and little knowledge on their influence had been available before Dr. Tang took up the subject in his Ph.D. research under the supervision of Professor M. C. Shen.

For the elucidation and prediction of such peristalsis, Profess Shen and Dr. Tang have developed two aymptotic methods which are fully nonlinear as needed in applications; one is designed for small Reynolds numbers and the other, for waves long compared to the duct diameter over the whole range of Reynolds number of practical interest.

In order to demonstrate the validity of these approximations conclusively, they have also established the rigorous theory of the nonlinear Overbeck-Boussinesq equations governing peristaltic fluid motion with heat conduction and heat release on a wide basis. By the help of the large apparatus of inequalities needed to make either degree theory or Galerkin limits work, they have proved the wellposedness of not only the basic periodic pumping state, but also of the problems for unsteady and nonperiodic motions of the duct walls. These theorems, moreover, have permitted them to prove the stability of the basic pumping state by energy estimates. It appears hard to avoid the conclusion that this research will stand as the definitive theoretical work on peristalsis with heat release and heat conduction.

In addition, Dr. Tang has employed the mathematical knowledge gained to compute both exact and approximate solutions of the Overbeck-Boussinesq equations and has shown thereby that his approximations have satisfactory accuracy over the whole range of practical interest. Any nuclear or medical engineering project can therefore use his asymptotic methods confidently to predict the parameter ranges over which peristaltic pumping will work without back-flow or significant pockets of trapped liquid. A number of mathematical papers for the publication of all this work are in preparation.

Research by S. M. Sun under the supervision of M. C. Shen

1. Viscous magnetic fluid flow down an inclined plane.

The problem deals with a viscous magnetic fluid flow down an inclined plane under the influence of an applied magnetic field parallel to the plane. The commonly used method is to find a critical Reynolds number for stability in the long wave limit via linearized equations. Our approach, instead, leads to a nonlinear evolution equation governing the development of a weakly nonlinear disturbance on the free surface of the magnetic fluid, and determines as well the stability of the disturbance by considering the change of type of a linear second order differential operator in the equation. It has been found that in the long wave limit the component of the applied magnetic field aligned with the direction of wave propagation has a stabilizing effect and the transverse component, however, has a destabilizing effect. Furthermore, a critical Reynolds number has been explicitly obtained. We have also studied the case when the linear operator is parabolic, that is, the equation assumes the form

$$u_t + uu_x - u_{yy} = 0 \quad ,$$

by a coordinate transformation. This equation subject to a prescribed initial condition was solved by means of the method of characteristics and in general the solution may break down eventually [1].

We also note that the work on the inhomogeneous Burgers equation for a viscous fluid flow down an inclined plane with a prescribed surface disturbance is in good progress. We attempt to relax some of the assumptions imposed on the prescribed disturbance for the existence of a pulse-like solution [2].

References

[1] Viscous magnetic fluid flow down an inclined plane - in preparation (with M. C. Shen).

[2] Pulse-like solution of the inhomogeneous Burgers equation - in preparation (with M. C. Shen).